STUDY OF HEAT AND MASS TRANSFER PROCESSES IN PLANE CHANNELS OF ARBITRARY SHAPE OCCUPIED BY A GRID (MESH)

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A method is proposed for finding the temperature and concentration fields associated with the flow of liquid in plane channels of arbitrary shape, having surface heat and mass sources at their walls.

Problems of heat and mass transfer associated with the flow of liquid in plane channels of arbitrary shape occupied by three-dimensional separator grids or meshes are encountered in a number of practical applications, such as that of chemical current sources. The complicated geometrical structure of the channels makes it practically impossible to obtain an exact solution of the hydrodynamic problem on the basis of the Navier-Stokes equation; thus the general problem of heat and mass transfer cannot be solved. If the channels are not very thick and the velocity of the liquid is low, and if, furthermore, the structure of the three-dimensional separator grid corresponds (within a reasonable approximation) to the structure of porous media, it is convenient to restrict consideration to the two-dimensional flow pattern of the velocity averaged over the channel cross section, and to use the Darcy law for analyzing the motion [1]. If the coordinate system is referred to the principal axes of the permeability tensor, the equations of motion in the Darcy approximation assume the form

$$u_x = \frac{\partial \psi}{\partial y} = -\frac{k_x}{\mu}\frac{\partial p}{\partial x}; \quad u_y = -\frac{\partial \psi}{\partial x} = -\frac{k_y}{\mu}\frac{\partial p}{\partial y}, \tag{1}$$

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial p}{\partial y}\right) = 0, \tag{2}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{k_y}\frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{k_x}\frac{\partial \psi}{\partial y}\right) = 0, \tag{3}$$

where u_x and u_y are the velocity components; p is the pressure; ψ is the stream function; k_x and k_y are the components of the permeability tensor along the principal axes x and y, depending on the thickness of the slot and the structure of the separator; μ is the dynamic viscosity. If the permeabilities k_x and k_y are constant, Eqs. (1) and (3) may be given the appearance of Laplace equations by expressing them in dimensionless form:

$$\bar{u}_x = \frac{\partial \bar{\psi}}{\partial \bar{y}} = -\frac{\partial \bar{p}}{\partial \bar{x}}; \quad \bar{u}_y = -\frac{\partial \bar{\psi}}{\partial \bar{x}} = -\frac{\partial \bar{p}}{\partial \bar{y}}, \tag{4}$$

$$\frac{\partial^2 \bar{p}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{p}}{\partial \bar{y}^2} = 0, \tag{5}$$

$$\frac{\partial^2 \bar{\psi}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} = 0. \tag{6}$$

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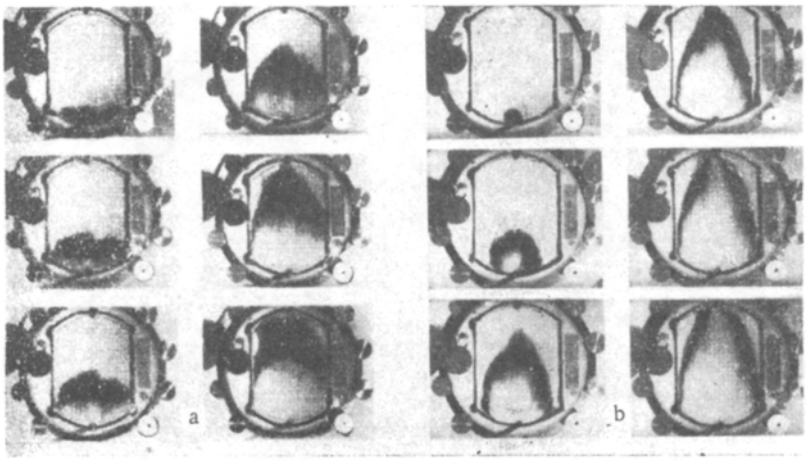

Fig. 1. Leading edges representing the motion of a tinted liquid through a channel of complex shape: a) extended entry; b) point entry.

Here $\bar{x} = x/X$; $\bar{y} = (y/X)\sqrt{k_x/k_y}$; $\bar{u}_x = (\rho X h\varepsilon/G)\sqrt{k_x/k_y}$; $\bar{u}_y = (\rho X h\varepsilon/G)u_y$; $\bar{\psi} = (\rho h\varepsilon/G)\psi$; $\bar{p} = (\rho h\varepsilon/\mu G)\sqrt{k_x k_y}p$, where X is the characteristic dimension in the x direction, h is the thickness of the slot, G is the rate of flow of the liquid through the chamber, ρ is the density of the liquid, and ε is the porosity of the separator.

The transformation so employed distorts the coordinate system and the velocity field in the same ratio

$$\frac{u_x}{u_y} = \sqrt{\frac{k_x}{k_y}\frac{\bar{u}_x}{\bar{u}_y}}; \quad \frac{x}{y} = \sqrt{\frac{k_x}{k_y}\frac{\bar{x}}{\bar{y}}}. \tag{7}$$

Since the walls of the channel are impermeable to the liquid, the boundary condition for Eq. (5) will be $\partial\bar{p}/\partial n = 0$. For Eq. (6) the stream function $\bar{\psi}$ is given at the boundary; the strengths of the sources and sinks have to be given at the points of liquid entry and exit and in the present steady-state problem their total strength should be zero.

Let us consider the problem of heat and mass transfer during the flow of a liquid in channels having surface heat or mass sources or sinks at their walls. Let us introduce these sources into the equations of heat and mass transfer, integrating the equations over the thickness of the slot. Neglecting the molecular transfer (heat conduction or diffusion) by comparison with convective transfer, we obtain an equation determining the change in the temperature or concentration averaged over the channel cross section:

$$\mathbf{u}\frac{dg}{ds} = \frac{\Omega(g)}{Wh\varepsilon}. \tag{8}$$

Here g denotes the temperature T or concentration c; Ω is a source depending on T or c; s is the direction along the stream line; W is a coefficient adjusting the dimensions of Eq. (8).

In Eq. (8) the variables are separated. Thus, if a particle of liquid passes into the channel at the point Q_0, having a temperature (or concentration) g_0, the temperature (concentration) of the liquid at the point Q may be found from the equation

$$\int_{g_0}^{g}\frac{dg}{\Omega(g)} = \frac{1}{Wh\varepsilon}\int_{Q_0}^{Q}\frac{ds}{\mathbf{u}}. \tag{9}$$

The integral on the right-hand side of (9) determines the time which elapses during the motion of the liquid from the point of entry Q_0 to the point Q and is calculated

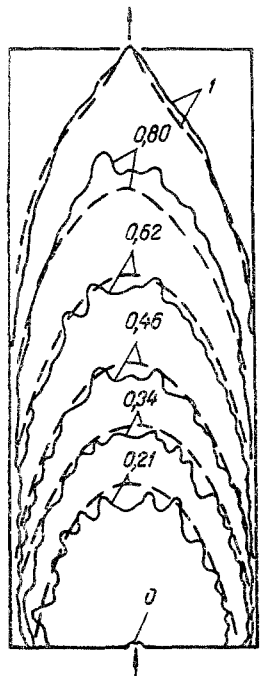

Fig. 2. Leading edges of liquid in a rectangular channel. Continuous curves derived from a model, broken curves by calculation.

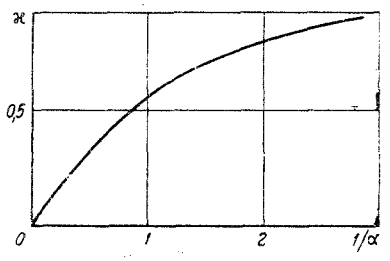

Fig. 3. Nonuniformity coefficient of the motion as a function of the geometry of the rectangular channel.

along the stream line. By using Eq. (7) we may transform to the dimensionless time of motion in the planes $\bar{x}$, $\bar{y}$:

$$\tau(Q_0, Q) = \int_{Q_0}^{Q} \frac{ds}{u} = \frac{\rho X \varepsilon h}{G} \sqrt{\frac{k_y}{k_x}}\, \bar{\tau}(\bar{Q}_0, \bar{Q}). \quad (10)$$

It follows from Eq. (9) that, if the density of the heat or mass sources depends solely on the temperature or concentration, the conditions at the entry remain constant, and molecular transfer processes are neglected, the isotherms or lines of equal concentrations coincide with the lines of equal times, i.e., the geometrical locus $\tau(Q_0, Q) = \text{const}$ of the points reached by the liquid particles after identical intervals of time. Thus, in order to calculate the temperature and concentration fields there is no need to know the velocity field; we only have to determine the integrated characteristics (lines of equal times), since every point in space is uniquely linked to the time of motion of the liquid particle from the point of entry, there being no vortices in the channel.

Let us consider two methods of determining the lines of equal times not requiring any determination of the velocity field: 1) a computing method, using conformal transformations; 2) a physical simulation technique. If the liquid enters and leaves the chamber through holes of small diameter d, so that $d/X \ll 1$, these may be regarded as points. In this case, in order to calculate the lines of equal times it is convenient to transform the plane $\bar{z} = \bar{x} + i\bar{y}$ conformally into the upper half plane $\text{Im}\ \omega \geq 0$, so that the point of entry Q_0 transforms to $\omega = 0$ and the exit point to $\omega = \infty$. The stream lines at the plane ω then become straight lines, and instead of calculating the curvilinear integral (10) we may integrate along straight stream lines. The solutions of Eqs. (5) and (6) in the ω plane take the form

$$\bar{p} = \text{const} - \frac{1}{2\pi} \ln R^2, \quad (11)$$

$$\bar{\psi} = \frac{1}{2} - \frac{\chi}{\pi}, \quad (12)$$

where R and χ are the polar coordinates in the plane ω. Using Eqs. (1), (4), (11), and (12), we obtain an expression for the dimensionless time of motion

$$\bar{\tau}(\bar{Q}_0, \bar{Q}) = \pi \int_0^{\varphi(\bar{Q})} |f'(R, \chi = \text{const})|^2 R\, dR, \quad (13)$$

where $\bar{z} = f(\omega)$; $\omega = \varphi(\bar{z})$.

By way of example, let us consider a channel of rectangular shape with a symmetrical entry and exit. In this case the conformal transformation is effected by means of an elliptical integral of the first kind $f(\omega) = cF(n, \omega)$ [2]. From Eq. (13) we obtain a formula for determining the line of equal times

$$\bar{\tau}(\bar{Q}_0, \bar{Q}) = \pi c^2 \int_0^{\varphi(\bar{Q})} \frac{RdR}{(1 - 2R^2\cos^2 2\chi + R^4)(1 - 2n^2R^2\cos^2 2\chi R^2 + n^4R^4)}, \quad (14)$$

where n and c are constants determined by the ratio of the sides of the rectangle $\alpha = (k_y/k_x)^{1/2}(X/Y)$.

The simulation method is effective in channels of complicated geometrical shape involving the extended entry and exit of the working substances, in which case finding the lines of equal times by calculation involves serious computing difficulties. The lines of equal times are determined by using a model of the channel made of transparent material, recording the progress of the leading edge of a tinted liquid in a motion-picture camera, and noting the corresponding times. A typical picture of the configurations of the lines at the leading edges in channel of complicated shape is presented in Fig. 1. The lines of equal times obtained by calculation from Eq. (14) are compared with those given by the simulation process in Fig. 2. We see from this figure that both methods lead to the same results, so confirming the validity of the assumptions made.

In order to provide a quantitative expression for the nonuniformity of the motion of the liquid in the chamber of a fuel cell, we introduce the nonuniformity coefficient of the motion $\varkappa$, equal to the ratio of the minimum time to the average time of motion of the liquid particles in the channel:

$$\varkappa = \frac{\tau_{min}}{\tau_{av}}. \quad (15)$$

Figure 3 shows the coefficient $\varkappa$ as a function of the ratio of the sides $1/\alpha$ for a rectangular channel. Clearly on the same principle we may introduce a coefficient for the nonuniformity of the temperature and concentration fields.

LITERATURE CITED

1. R. Lyczkowski and D. Gidaspow, "Two-dimensional inert accumulation in space hydrogen-oxygen fuel cells," AIChE Journal, 17, No. 5 (1971).
2. B. A. Fuks and B. V. Shabat, Functions of a Complex Variable and Some of Their Applications [in Russian], Nauka (1964).